



ASYMPTOTIC METHODS ESPECIALLY IN COMBUSTION

FINAL REPORT

G.S.S. LUDFORD

NOVEMBER 1981

U.S. ARMY RESEARCH OFFICE

GRANTS DAAG29-77-G-0210, -78-G-0134; CONTRACT DAAG-29-79-C-0121

CORNELL UNIVERSITY

APPROVED FOR PUBLIC RELEASE: DISTRIBUTION UNLIMITED



The view, opinions, and/or findings contained in this report are those of the authors and should not be construed as an official Department of the Army position, policy, or decision, unless so designated by other documentation.

DIE FILE COPY

THE PARTY OF A

UNCLASSIFIED
SECURITY CLASSIFICATION OF THIS PAGE (When Date Entered)

REPORT DOCUMENTATION PAGE	READ INSTRUCTIONS BEFORE COMPLETING FORM
T	3 RECIPIENT'S CATALOG NUMBER
NA AIDP	023
4 TITLE (and Subsissio)	TYPE OF REPORT & PERIOD COVERED
Asymptotic Methods Especially in Combustion	/ Final 15 Sept./77-14 June/81
	6 PERFORMING ORG REPORT NUMBER
7. AUTHOR(e)	B. CONTRACT OR GRANT NUMBER(+)
()G.S.S. Ludford ()	DAAG29- {77-G-0210 78-G-0134 79-C-0121
9. PERFORMING ORGANIZATION NAME AND ADDRESS	10 PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
Cornell University 018	P-14933-M
Ithaca, NY 14853 0/0	P÷15882-M
11. CONTROLLING OFFICE NAME AND ADDRESS	12. REPORT DATE
U. S. Army Research Office	November 1981
rost printe box 12211	November 1981
Research Triangle Park, NC 27709	
14. MONITORING AGENCY NAME & ADDRESS(If different from Controlling Office)	15. SECURITY CLASS. (of thie report)
	Unglassified
	15a. DECLASSIFICATION DOWNGRADING SCHEDULE NA
IS DISTRIBUTION STATEMENT (of this Report)	
17 DISTRIBUTION STATEMENT for the ebstract entered in Black 20, If different from Report)	
TI A	
18 SUPPLEMENTARY NOTES	
The view, opinions, and/or findings contained in this report are those of the author(s) and should not be construed as an official Department of the Army position, "olicy, or decision, unless so designated by other documentation.	
Asymptotic methods, combustion, quenching, DDT, near-stoichiometry, propagation and flammability limits, diffusion flames, thermal runaway and explosion, ignition and extinction, deflagrations and detonations, flame tips, flame stretch, excessenthalpy flames, dissociation, droplet burning, shear and strain, differential mass diffusion, surface equilibrium.	
Modern asymptotic methods have been applied to a wide range of problems in combustion science as well as a few in magnetohydrodynamic and fluid mechanical questions. Details are contained on the 39 Technical Reports, 6 Ph.D. Theses, and 1 Master's Thesis listed. A list of participating scientists is given.	

DD , " ORM 1473

EDITION OF ! NOV 65 IS OBSOLETE

UNCLASSIFIED

J. 10.

Objectives and Results

SACA MANAGEMENT

Asymptotic methods were applied to a variety of combustion problems including

- (i) quenching, in all its aspects,
- (ii) the deflagration to detonation transition (DDT),
- (iii) near-stoichiometric behavior of two-reactant mixtures,
 - (iv) propagation limits of mixtures,
 - (v) chambered diffusion flames,
 - (vi) thermal runaway in two-reactant systems.
- (vii) burning of droplets.

The most important results may be summarized as follows.

- (a) Mathematically rigorous treatment of thermal runaway.
- (b) Description of chambered diffusion flames for different supply temperatures.
- (c) A theory of quenching in shear flows.
- (d) Discovery that differential diffusion can account for shift in flame-velocity maximum of mixtures; and demonstration that dissociation plays a minor role, as does the dilution of the mixture.
- (e) Commonly accepted ignition and extinction criteria justified by a stability analysis.
- (f) Rational analysis of flame tips.
- (g) Limitations of accepted droplet theory; and its invalidity for variations via the ambient pressure.
- (h) Simple and comprehensive description of detonation structure.
- (i) Existence and non-existence criteria for the structure of detonation waves.
- (j) Analysis of the concept of flame stretch.
- (k) Potential uses of activation-energy asymptotics in turbulent combustion theory.
- (1) Description of fast deflagration waves, whose theory is outside the combustion approximation; and the discovery of very slow deflagrations.

- (m) Incorporation of fluid mechanics into the problems of flame tips and stretch.
- (n) Theory of quasi-steady flame acceleration and deceleration, as an aspect of DDT.
- (o) Stability analysis of near extinction for the burning fuel drop.
- (p) Demonstration of stable steady states for flames held in the stagnation-point flow behind a wire or bluff body.
- (q) Theory of the excess-enthalpy flame.
- (r) Theory of flammability limits.

Most of these results have been incorporated into a book entitled "Theory of Laminar Flames", written by J. Buckmaster and G.S.S. Ludford for the Cambridge Monograph Series on Mechanics and Applied Mathematics.

In addition a few problems in magnetohydronamics and fluid mechanics have been addressed.

Details of all the results are contained in Progress Reports Nos. 25-32.

Publications

A STATE OF

- 39 Technical Reports were written, as follows.
- 90. J.D. Buckmaster: Combustion at a stagnation point.
- 91. M. Matalon & G.S.S. Ludford: The asymptotic derivation of isola and S responses for chambered diffusion flames. SIAM Journal on Applied Mathematics, 37, (1979), 107.
- 92. J. Buckmaster: The quenching of a deflagration wave held in front of a bluff body. Proceedings of the Seventeenth Symposium (International) on Combustion, Leeds 1978, p. 835. (Published by The Combustion Institute, 1979.) Also presented at the Eighth U.S. National Congress of Applied Mechanics, Los Angeles 1978.
- 93. G.S.S. Ludford & J.S. Walker: <u>Current status of MHD duct flow.</u>
 Proceedings of the 2nd Bat-Sheva International Seminar on MHD-Flows and Turbulence, Beersheva 1978, p. 83. (Published by Israel Universities Press, 1980.) Invited paper.
- 94. J. Buckmaster: A mathematical description of open and closed flame tips. Combustion Science and Technology 20 (1979), p. 287.
- 95. M. Matalon & G.S.S. Ludford: <u>Chambered diffusion flames for different supply temperatures</u>. Acta Astronautica <u>6</u> (1979), p. 1531. Presented at the Eighth U.S. National Congress of Applied Mechanics, Los Angeles 1978.

96. J.S. Walker & G.S.S. Ludford: <u>Liquid-metal flows in open channels</u>. Proceedings of the 2nd Bat-Sheva International Seminar on MHD-Flows and Turbulence, Beersheva 1978, p. 97. (Published by Israel Universities Press, 1980.)

THE PARTY OF

- 97. M.J. Normandia and G.S.S. Ludord: Surface equilibrium in drop combustion. SIAM Journal on Applied Mathematics 38 (1980), p. 326.
- 98. A.K. Sen & G.S.S. Ludford: The near-stoichiometric behavior of combustible mixtures. Part I: Diffusion of the reactants. Combustion Science and Technology 21 (1979), p. 15.
- 99. Moshe Matalon & G.S.S. Ludford: On the near-ignition stability of diffusion flames. International Journal of Engineering Science 18 (1980) p. 1017.
- 100. G.S.S. Ludford & J. Buckmaster: Activation-energy asymptotics in turbulent combustion. Proceedings of the SIMS Conference on Fluid Mechanics in Energy Conversion, Alta (Utah) 1979, p. 263. (Published by SIAM, 1980.) Invited paper.
- 101. G.S.S. Ludford & Asok K. Sen: Burning rate maximum of a plane premixed flame. Proceedings of the Seventh International Colloquium on Gasdynamics of Explosions and Reactive Systems, Göttingen 1979. Progress in Astronautics and Aeronautics, 76 (1981). p. 427. (Combustion in Reactive Systems, ed. by J. Ray Bowen, N. Manson, Antoni K. Oppenheim, and R.I. Soloukhin.)
- 102. G.S.S. Ludford: <u>Premixed cylindrical flames</u>. Transactions of the 25th Conference of Army Mathematicians, Baltimore (Md.) 1979, p. 155. (Published by ARO, Report 80-1.)
- 103. J. Buckmaster: Two examples of a stretched flame. To appear in the Quarterly Journal of Mechanics and Applied Mathematics.
- 104. L.-C. Li & G.S.S. Ludford: The overshoot in entry flow. To appear in Archives of Mechanics 32 (1980), 741. Presented at the XIV Biennial Fluid Dynamics Symposium, Posnan (Poland), 1979.
- 105. Philip Holmes and D.S. Stewart: The existence of one dimensional, steady detonation waves in a simple model problem. To appear in Studies in Applied Mathematics.
- 106. Asok K. Sen & G.S.S. Ludford: Effects of mass diffusion on the burning rate of non-dilute mixtures. Proceedings of the Eighteenth Symposium (International) on Combustion, Waterloo 1980, p. 417. (Published by The Combustion Institute, 1981.) Also an invited paper at the International Conference on Physico-Chemical Hydrodynamics, Madrid (Spain) 1980.
- 107. J.D. Buckmaster & G.S.S. Ludford: Mathematical theory of laminar combustion
 V: Unsteady burning of a linear condensate. To appear in "Theory of
 Laminar Flames", Cambridge Monographs on Mechanics and Applied Mathematics.

- 108. J.D. Buckmaster & G.S.S. Ludford: Mathematical theory of laminar combustion VI: Spherical diffusion flames. To appear in "Theory of Laminar Flames", Cambridge Monographs on Mechanics and Applied Mathematics.
- 109. J.D. Buckmaster & G.S.S. Ludford: <u>Mathematical theory of laminar</u> combustion VII: Cylindrical and spherical premixed flames. To appear in "Theory of Laminar Flames", Cambridge Monographs on Mechanics and Applied Mathematics.

A LANGE

- 110. J.D. Buckmaster & G.S.S. Ludford: Mathematical theory of laminar combustion VIII: Three-dimensional flames. To appear in "Theory of Laminar Flames", Cambridge Monographs on Mechanics and Applied Mathematics.
- 111. J.D. Buckmaster & G.S.S. Ludford: Mathematical theory of la minar combustion IX: Burner flames. To appear in "Theory of Laminar Flames", Cambridge Monographs on Mechanics and Applied Mathematics.
- 112. J.D. Buckmaster & G.S.S. Ludford: Mathematical theory of laminar combustion X: Effects of shear and strain. To appear in "Theory of Laminar Flames", Cambridge Monographs on Mechanics and Applied Mathematics.
- 113. J.D. Buckmaster & G.S.S. Ludford: <u>Mathematical theory of laminar combustion XI: Stability</u>. To appear in "Theory of Laminar Flames", Cambridge Monographs on Mechanics and Applied Mathematics.
- 114. J.D. Buckmaster & G.S.S. Ludford: <u>Mathematical theory of laminar combustion</u>
 XII: <u>Ignition and explosion</u>. To appear in "Theory of Laminar Flames",

 Cambridge Monographs on Mechanics and Applied Mathematics.
- 115. R.O. Ayeni & G.S.S. Ludford: On uniqueness of positive solutions of nonlinear elliptic boundary-value problems. To appear in the Nigerian Journal of Natural Sciences.
- 116. R.O. Ayeni & G.S.S. Ludford: Nonexistence of global solutions of a boundary-value problem of parabolic type. To appear in the Nigerian Journal of Natural Sciences.
- 117. G.C. Lu & G.S.S. Ludford: Asymptotic analysis of plane steady detonations. To appear in the SIAM Journal on Applied Mathematics.
- 118. D.S. Stewart & G.S.S. Ludford: <u>Fast deflagration waves</u>. Submitted for publication.
- 119. R.O. Ayeni & G.S.S. Ludford: <u>Rigorous bounds on temperature of unsteady</u> diffusion flames. Submitted for publication.
- 120. G.S.S. Ludford & D.S. Stewart: Mathematical questions from combustion theory. Transactions of the 26th Conference of Army Mathematicians, Lebanon (NH) 1980, p. 53. (Published by ARO, Report 81-1.)

- 121. J.Buckmaster & T. Takeno: Blow-off and Flashback of an excess enthalpy flame. To appear in Combustion Science and Technology.
- 122. D.S. Stewart & G.S.S. Ludford: <u>Deflagration and detonation for small heat</u> release. Submitted for publication.
- 123. J. Buckmaster & D. Mikolaitis: A flammability-limit model for upward propagation through lean methane/air mixtures in a standard flammability tube. To appear in Combustion and Flame.
- 124. J. Buckmaster & A. Nachman: <u>Propagation of an unsteady flame in a duct of varying cross-section</u>. To appear in Quarterly Journal of Mechanics and Applied Mathematics.
- Asok K. Sen & G.S.S. Ludford: The near-stoichiometric behavior of combustible mixtures. Part II: Dissociation of the products. To appear in Combustion Science and Technology.
- 126. J. Buckmaster & D. Mikoliatis: Flame stabilization in a rear stagnation point flow. Submitted for publication.
- 127. G.S.S. Ludford & D.S. Stewart: A theory of deflagrations and its application. Proceedings of the First Specialists Meeting (International) of the Combustion Institute, Bordeaux (France), July 1981. Invited paper.
- 128. J. Buckmaster & D. Mikolaitis: The premixed flame in a counterflow. Submitted for publication.
 - 6 Ph.D. theses were completed, as follows.
- Peuben Clafenwa Ayeni: Thermal runaway, v + 76 pp., May 1978.
- Michael Joseph Normandia: Mathematical theory of liquid fuel drops and monopropellants using the Clausius-Clapeyron relation, vii + 173 pp., May 1978.
- Samuel Paolucci: Langmuir circulations as a convective instability mechanism and its effect on the ocean mixed layer, x + 218 pp., May 1979.
- Asok Kumar Sen: Asymptotic analysis of near-stoichiometric flame propagation, vi + 123 pp., August 1979.
- Karen Ann Ames: Comparison results for related properly and improperly posed Cauchy problems, with application to mechanics, vi + 156 pp., August 1980.
- Donald Scott Stewart: The transition from deflegration to detonation, xii + 193 pp., May 1981.

In addition there was 1 Master's thesis, namely

Luen-Chau Li: Overshoot in the velocity profile for flow past a semi-infinite flat plate, v + 43 pp., January 1979.

Participating Scientists.

K.A. Ames (Ph.D.), C. Holmes (M.S.), H.V. McConnaughey, R.D.Janssen, L.C. Li (M.S.), G.C. Lu, A. Oyediran, A.K. Sen (Ph.D.), J.A. Simmen (M.S.) and D.S. Stewart (Ph.D.) were supported for various periods as Research Assistants. Advanced degrees that they earned are shown in parentheses. In addition R.O. Ayeni and S. Paolucci, who were supported at earlier times, earned Ph.D.'s during the report period.

M. Matalon and D.S. Stewart had postdoctoral appointments. Prof. J.D. Buckmaster of the University of Illinois has been a consultant every summer.

